

Overcoming the Limitations of Localization Uncertainty

Efficient & Exact Non-Linear Post-Processing and Calibration

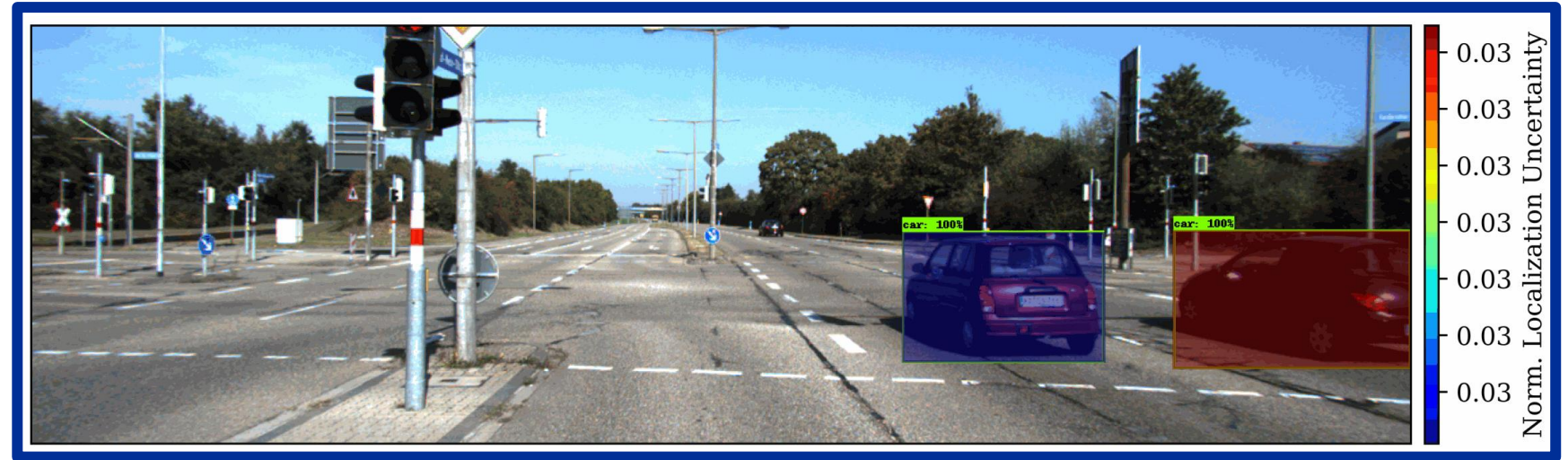
Moussa Kassem Sbeyti, Michelle Karg, Christian Wirth, Azarm Nowzad and Sahin Albayrak

Results

Visualization – KITTI

Top 10 frames with
lowest and highest
uncertainty out of
100 frames

*Low uncertainty for
close and clear objects*



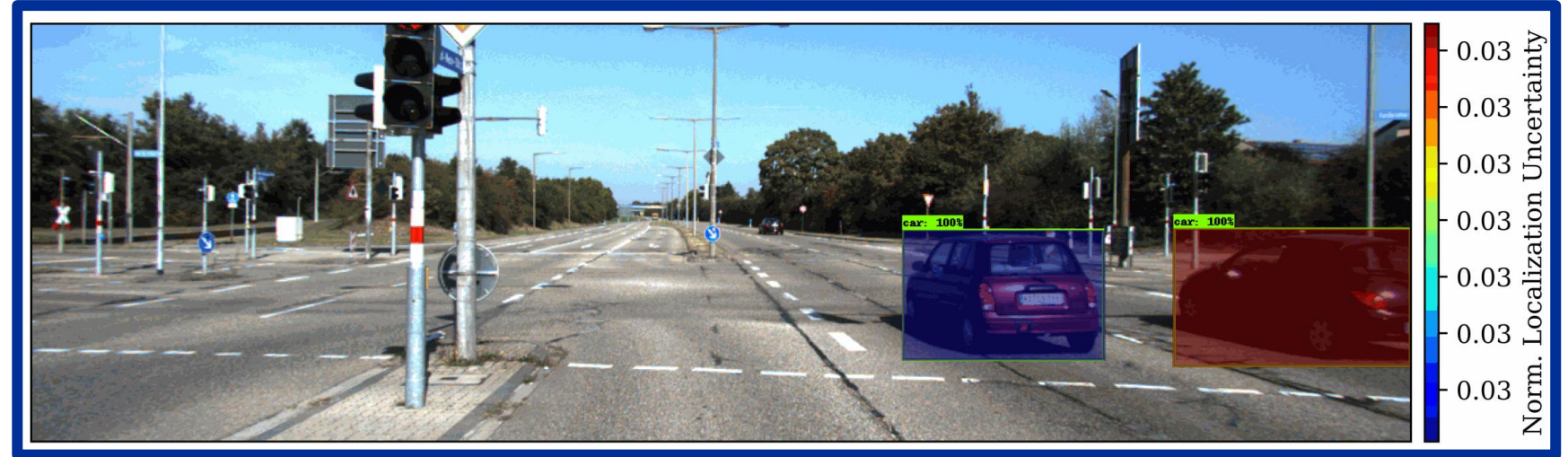
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Visualization – KITTI

Top 10 frames with **lowest** and **highest** uncertainty out of 100 frames

Low uncertainty for close and clear objects

High uncertainty for far, occluded and poorly detected objects



Results

Visualization – BDD100K

Top 10 frames with **lowest** and **highest** uncertainty out of 100 frames

Low uncertainty for close and clear objects



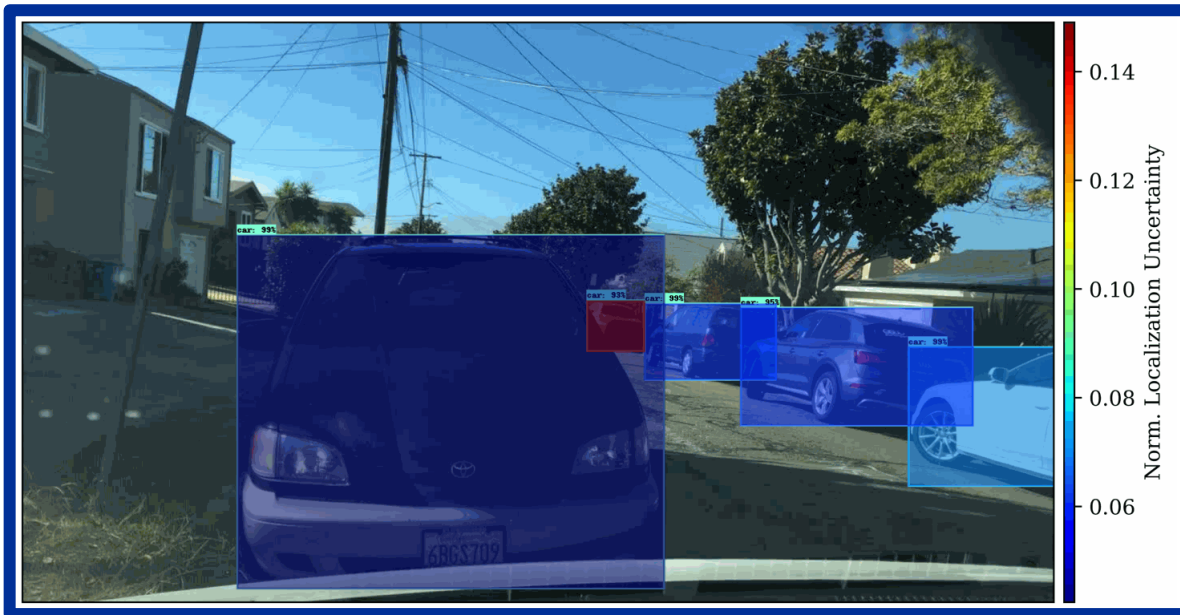
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Visualization – BDD100K

Top 10 frames with **lowest** and **highest** uncertainty out of 100 frames

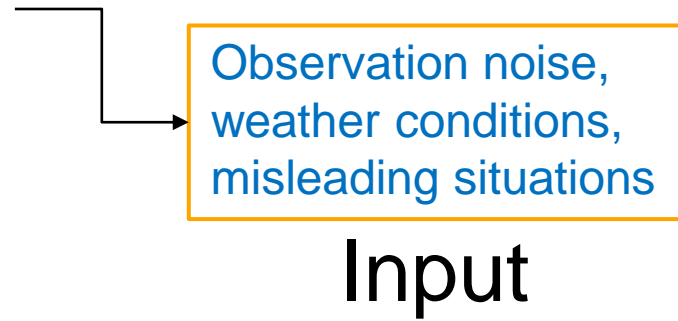
Low uncertainty for close and clear objects

High uncertainty for far, occluded and poorly detected objects



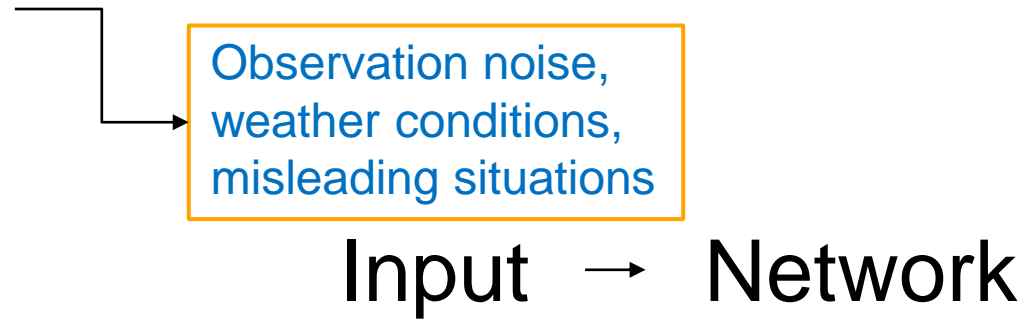
Loss Attenuation

Aleatoric Uncertainty



Loss Attenuation

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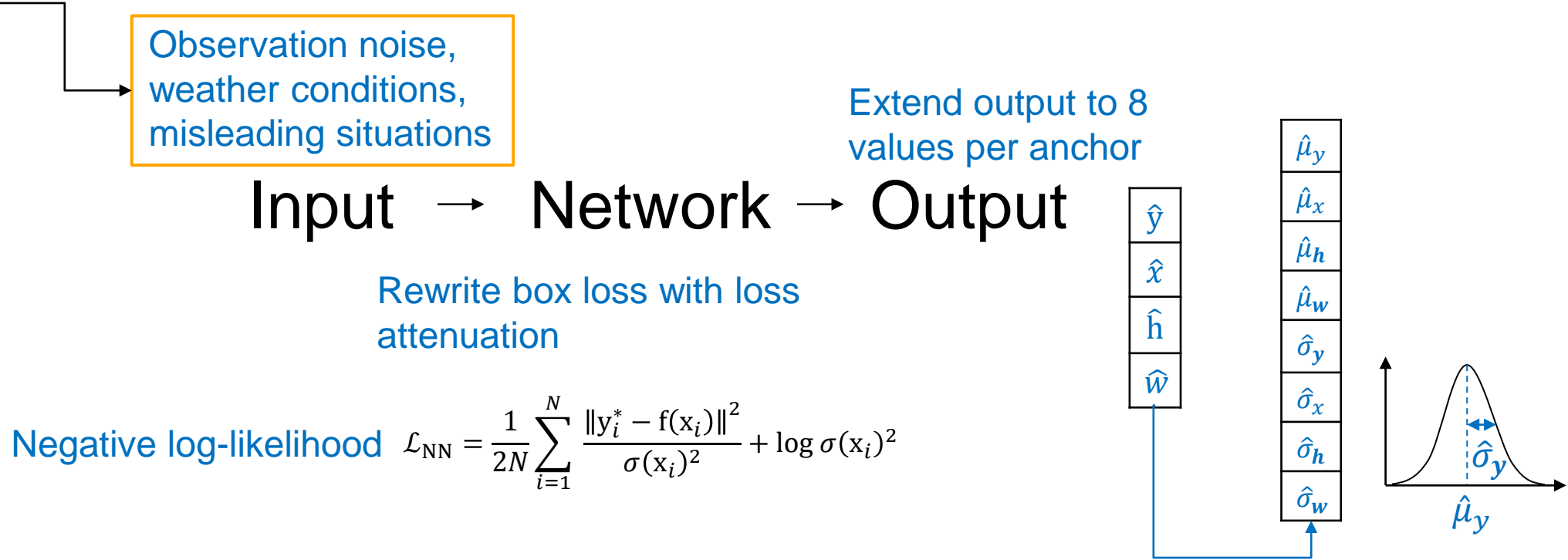


Rewrite box loss with loss
attenuation

Negative log-likelihood $\mathcal{L}_{\text{NN}} = \frac{1}{2N} \sum_{i=1}^N \frac{\|y_i^* - f(x_i)\|^2}{\sigma(x_i)^2} + \log \sigma(x_i)^2$

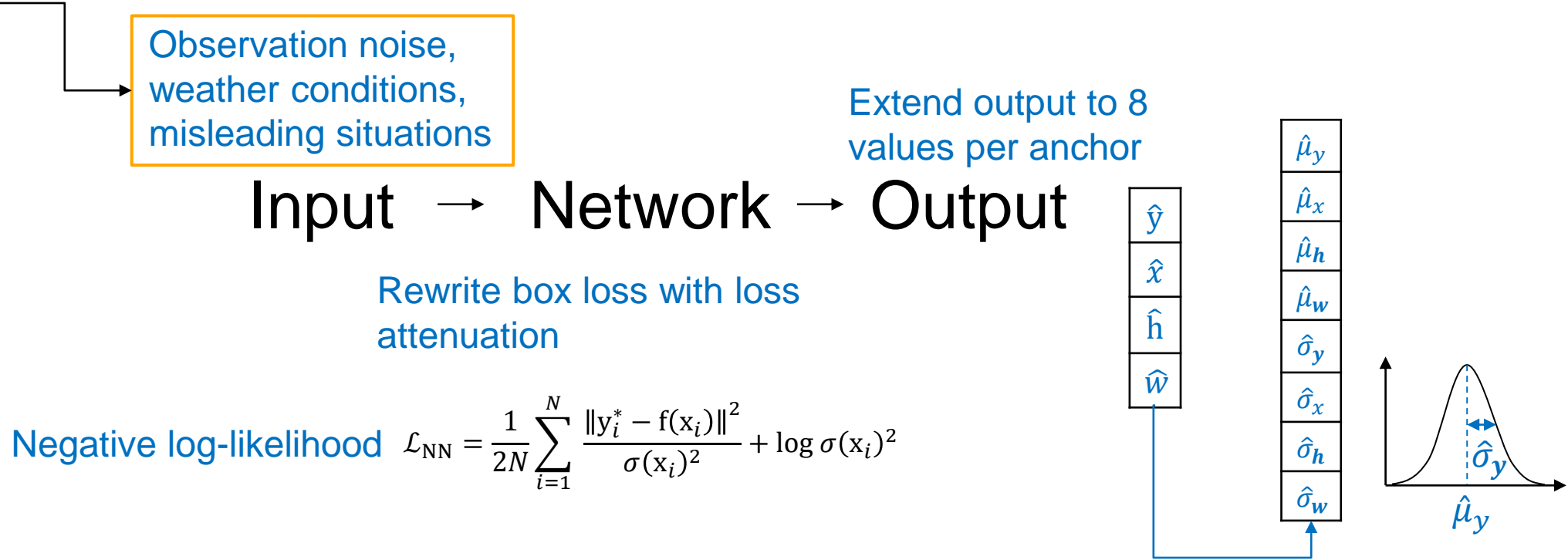
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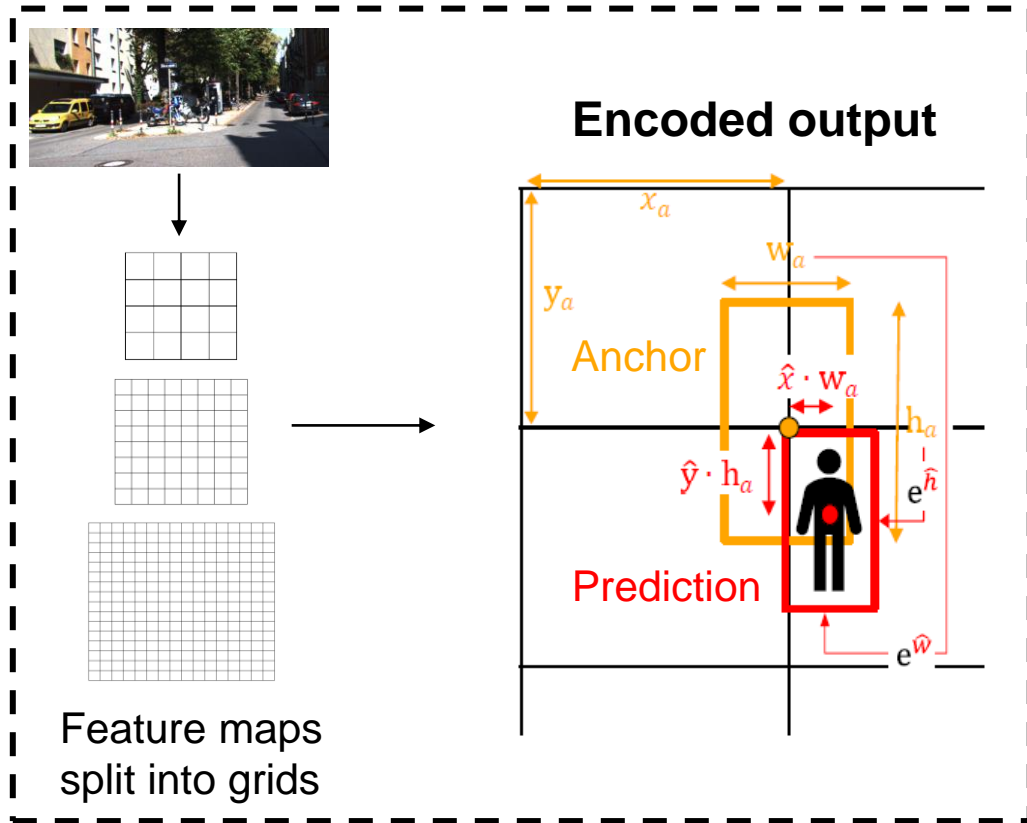


The uncertainty is learned as a function of the data

Use Case Definition

How is the output distribution $\mathcal{N}(\mu, \sigma^2)$ propagated through non-linear functions?

Network Output

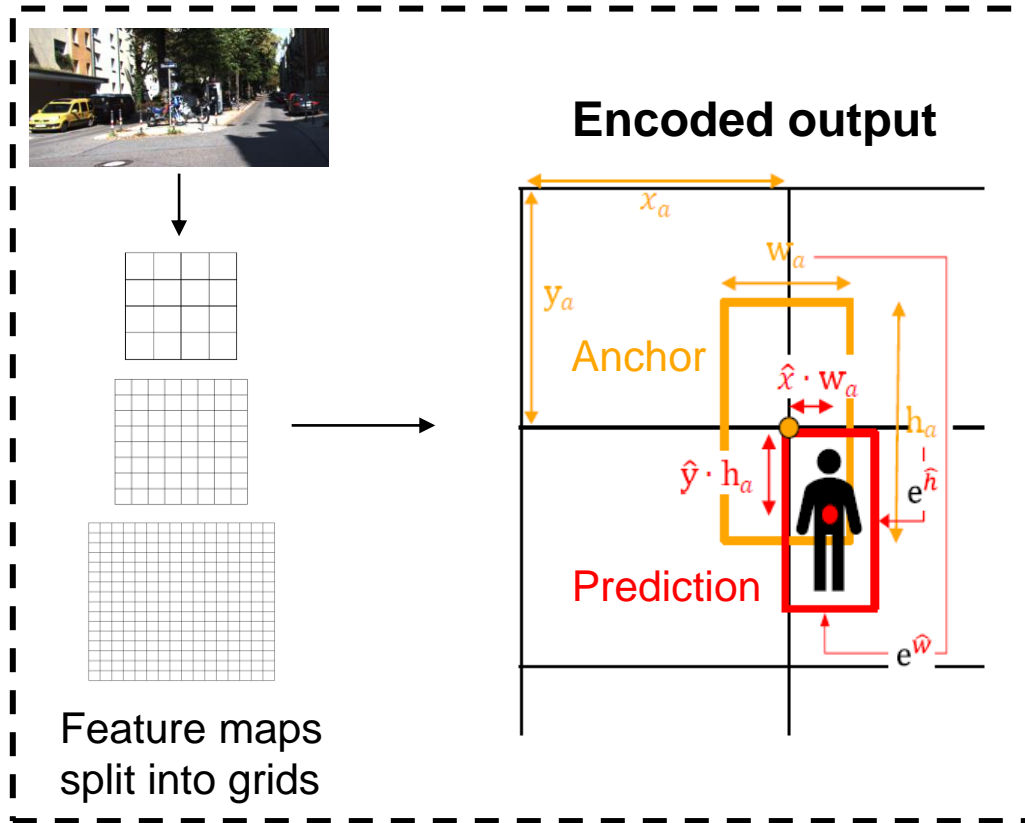


Anchor-relative center coordinates (\hat{x}, \hat{y}) , width \hat{w} and height \hat{h} .
Anchor center coordinates (x_a, y_a) , width w_a and height h_a .

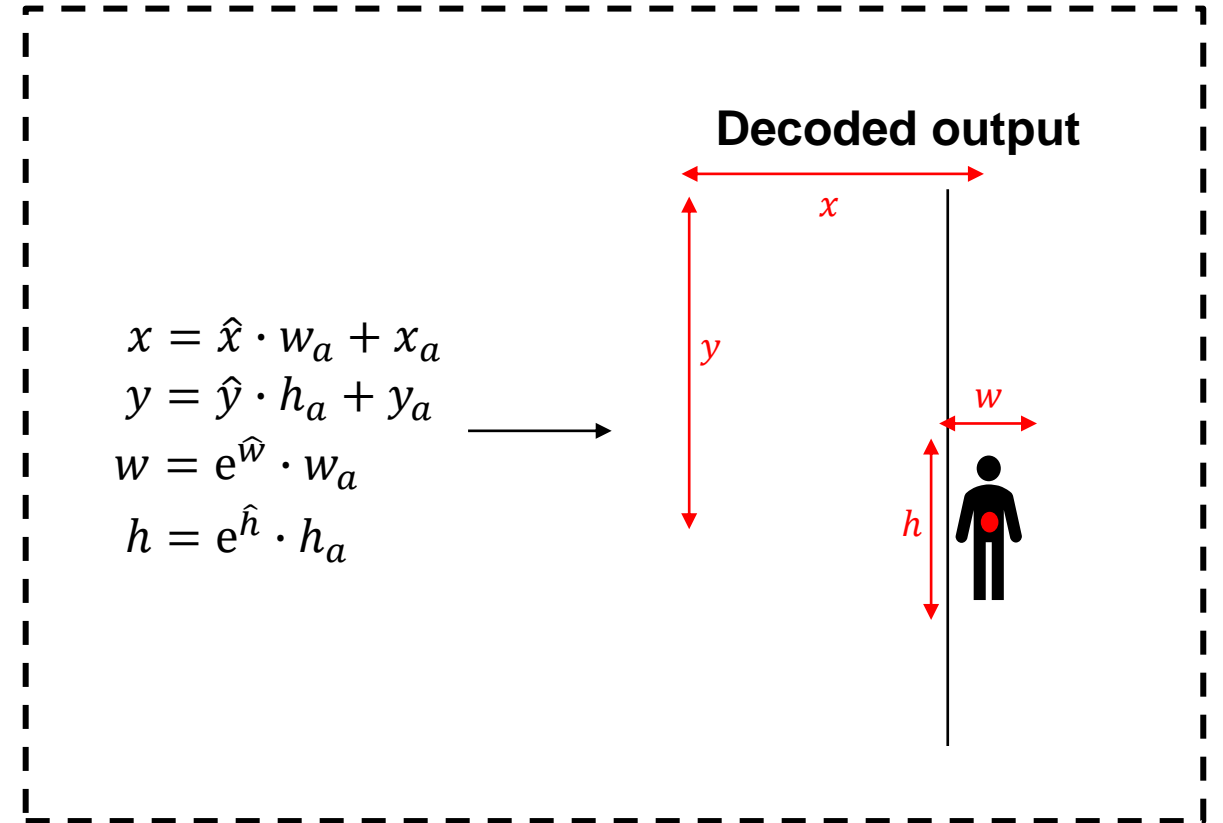
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Post-processing

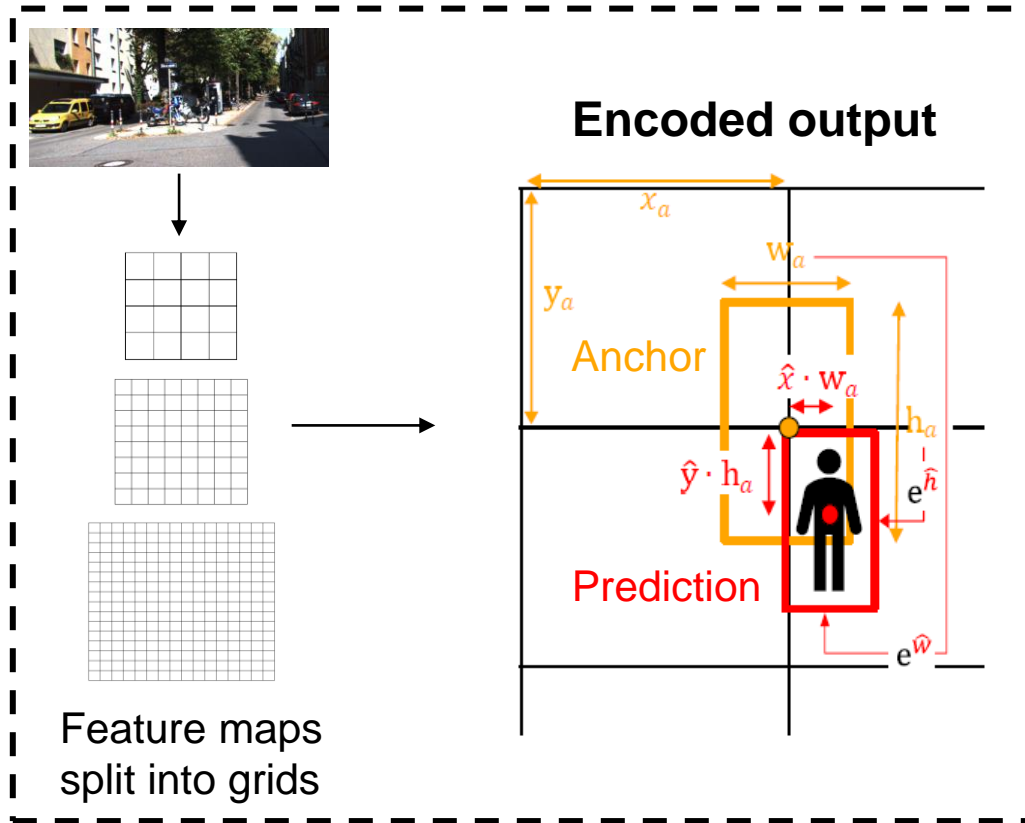


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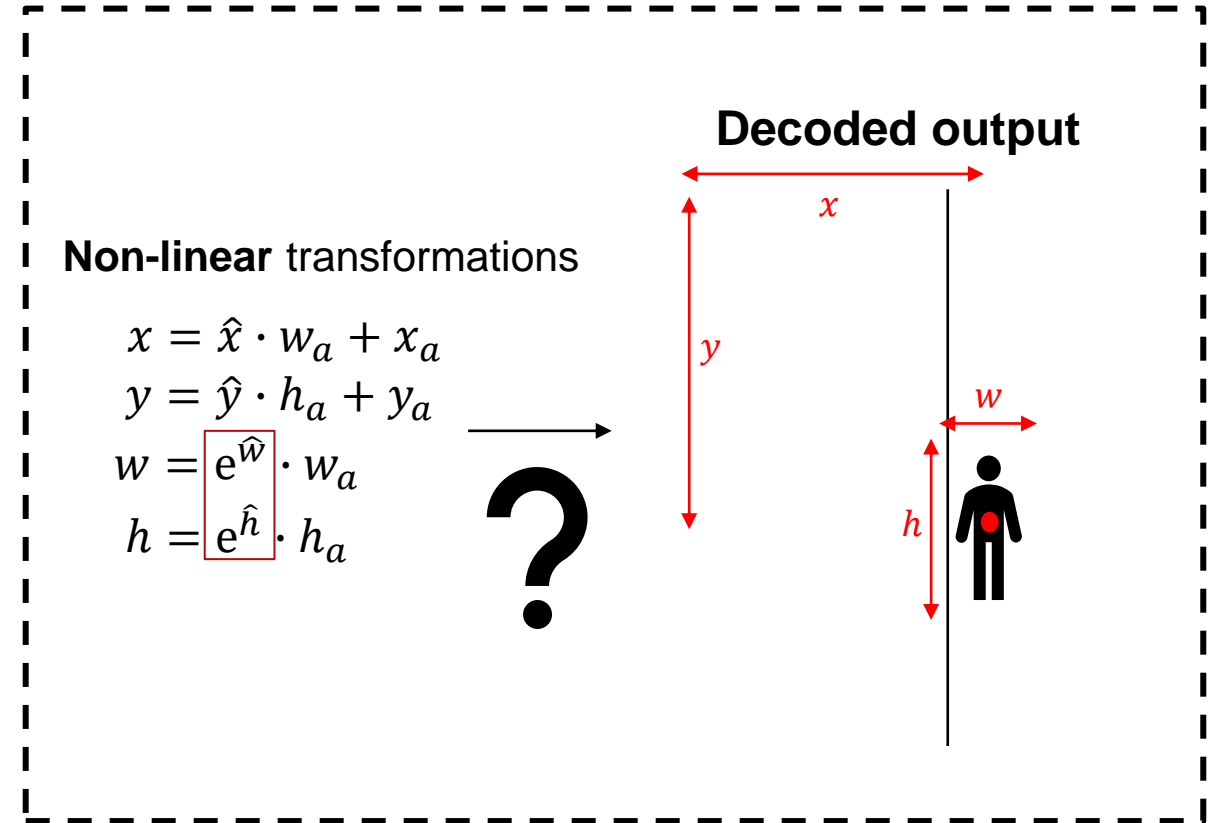
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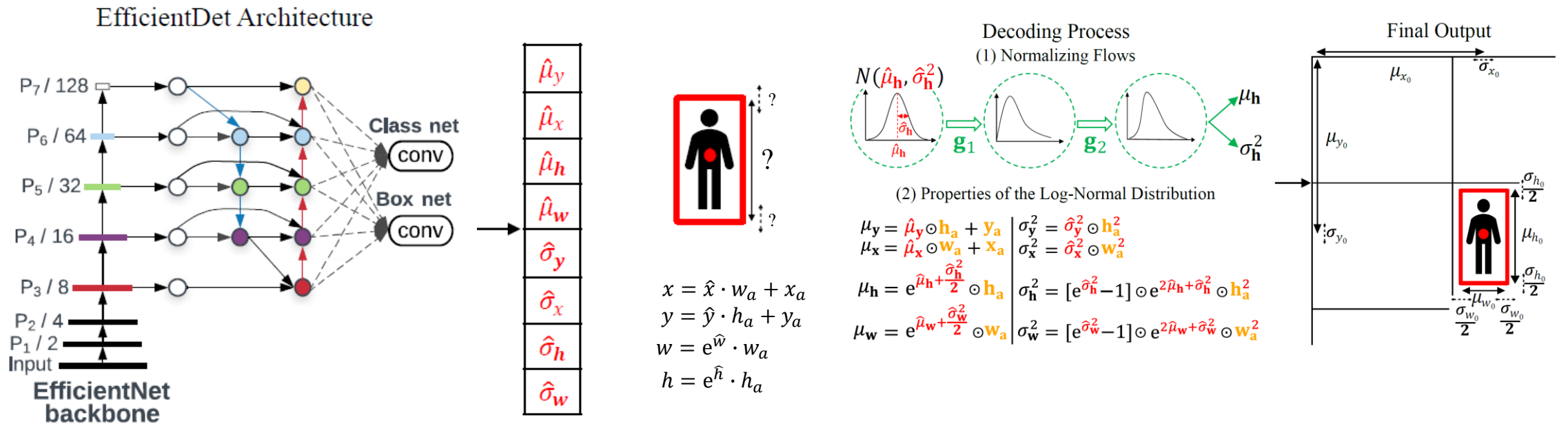


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Propagation Methods

Illustration

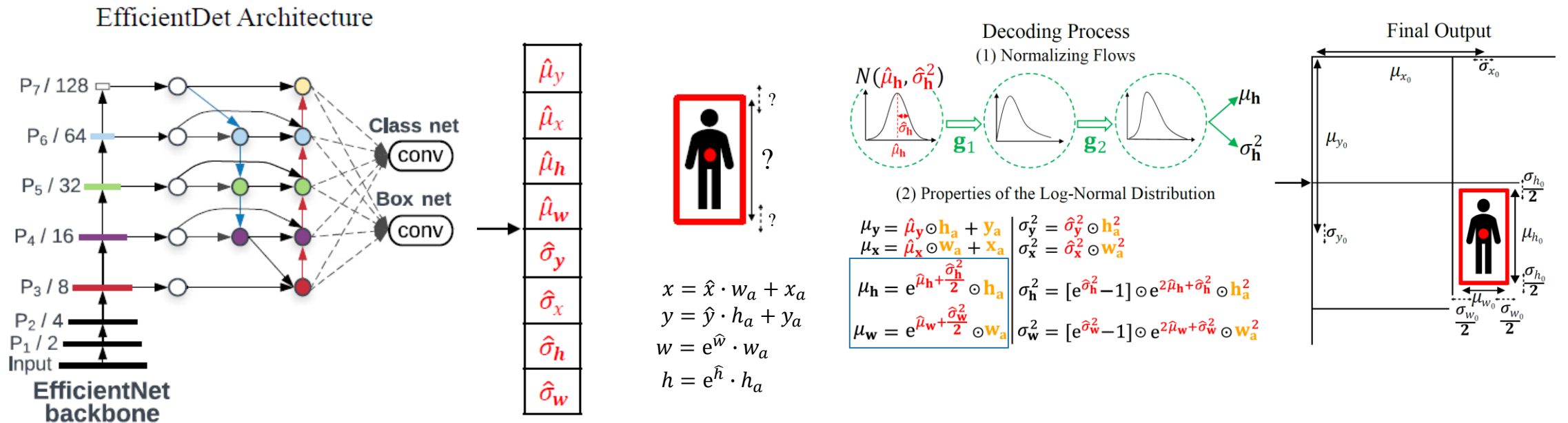
Loss attenuation in EfficientDet $\mathcal{L}_{NN} = \frac{1}{2 \cdot 4N_{pos}} \sum_{i=1}^N \sum_{j=1}^4 \left(\frac{\|y_{ij}^* - \hat{\mu}_j(\mathbf{x}_i)\|^2}{\hat{\sigma}_j(\mathbf{x}_i)^2} + \log \hat{\sigma}_j(\mathbf{x}_i)^2 \right) \cdot \mathbf{m}(y_{ij}^*)$



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Results

Propagation Methods

- › **EfficientDet-D0** pre-trained on **COCO**
- › Input resolution: **1024x512**
- › Soft-NMS output is **reordered** based on **lowest MSE**
- › **BDD100K**: 10 classes
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Baseline	72.8 ± 0.1	5.07 ± 0.1	90.1 ± 0.1	-	115.6 ± 3	34.8 ± 4
FalseDec	73.1 ± 0.5	5.27 ± 0.1	90.3 ± 0.1	4.27 ± 0.1	116.0 ± 3	31.1 ± 3
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Increases localization performance

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Faster than baseline and sampling*

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N-flow	73.3 ± 0.5	5.17 ± 0.2	90.3 ± 0.0	3.22 ± 0.0	116.6 ± 1	31.6 ± 3
Samp30	68.6 ± 0.4	5.43 ± 0.1	88.7 ± 0.1	3.19 ± 0.0	118.8 ± 2	34.5 ± 3
Samp100	71.8 ± 0.5	5.23 ± 0.1	90.1 ± 0.0	3.20 ± 0.0	117.4 ± 4	47.0 ± 3
Samp1000	73.1 ± 0.5	5.18 ± 0.2	90.4 ± 0.0	3.21 ± 0.0	117.9 ± 4	187.4 ± 4
Baseline	24.7 ± 0.1	8.96 ± 0.2	66.6 ± 1.6	-	115.7 ± 3	33.0 ± 4
FalseDec	23.9 ± 0.2	8.81 ± 0.2	67.3 ± 0.0	4.40 ± 0.1	115.9 ± 2	30.4 ± 4
L-norm	24.4 ± 0.1	8.53 ± 0.2	67.7 ± 0.0	3.69 ± 0.0	115.3 ± 1	30.6 ± 4
N-flow	24.4 ± 0.1	8.53 ± 0.2	67.7 ± 0.0	3.69 ± 0.0	116.4 ± 1	31.0 ± 3
Samp30	21.0 ± 0.1	9.02 ± 0.2	64.7 ± 0.0	3.70 ± 0.0	118.0 ± 3	33.6 ± 4
Samp100	23.2 ± 0.1	8.68 ± 0.2	66.7 ± 0.0	3.69 ± 0.0	117.0 ± 3	45.4 ± 4
Samp1000	24.2 ± 0.1	8.55 ± 0.2	67.6 ± 0.1	3.69 ± 0.0	118.4 ± 3	187.3 ± 4

KITTI

BDD

AP: Average precision

RMSE: Root-mean-square error

mIoU: Average intersection over union

NLL: Negative log-likelihood

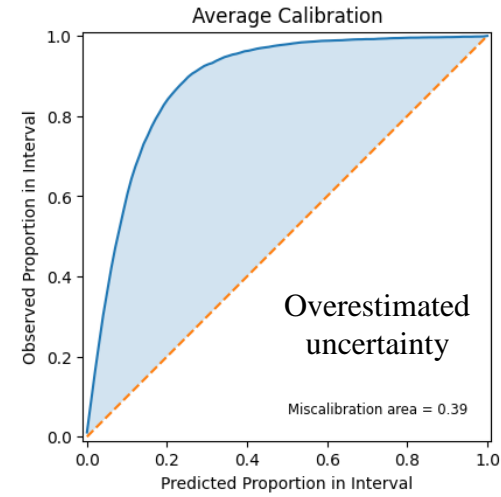
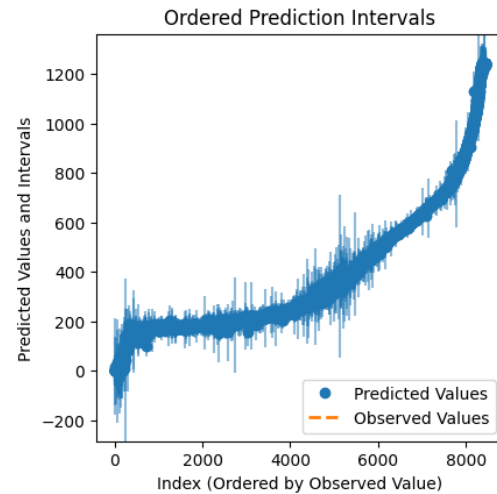
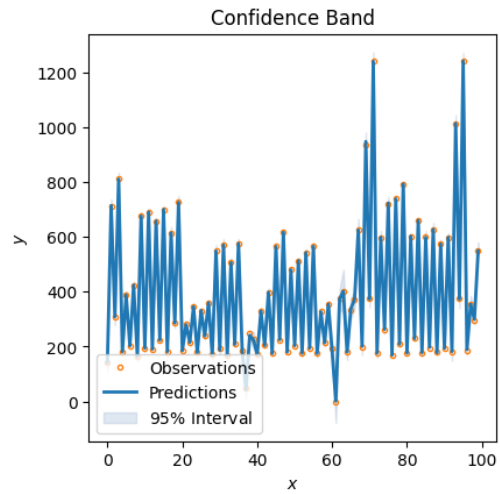
ET: Model exporting time in seconds

IT: Inference time in milliseconds

*Increases localization performance
Faster than baseline and sampling
Addresses the drawbacks of sampling*

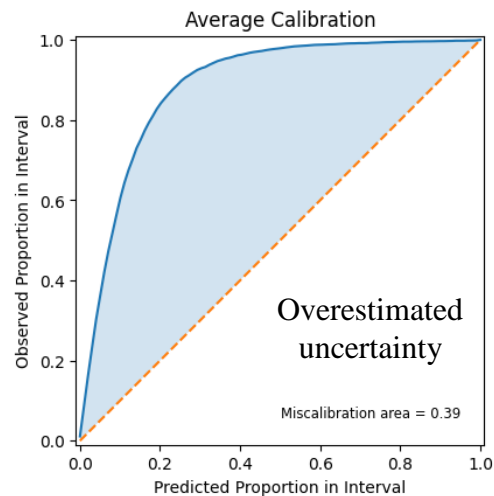
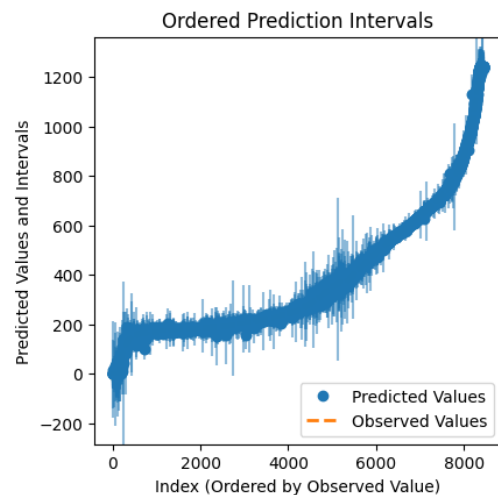
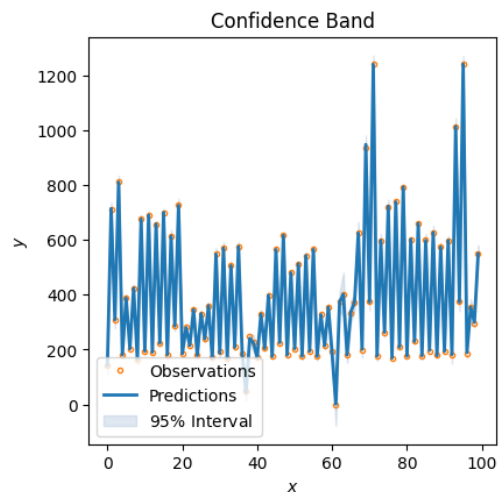
Calibration

Is the predicted uncertainty well-calibrated?

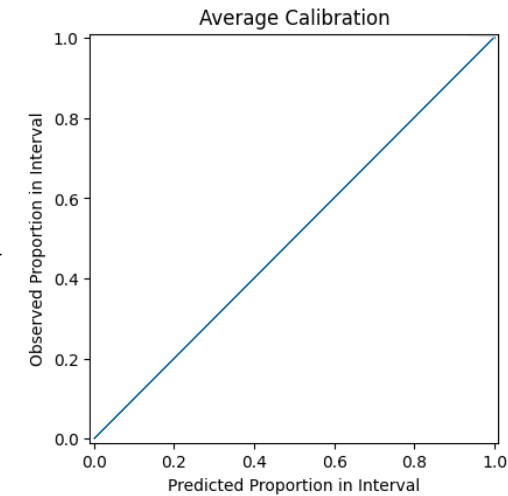


Calibration

Is the predicted uncertainty well-calibrated?

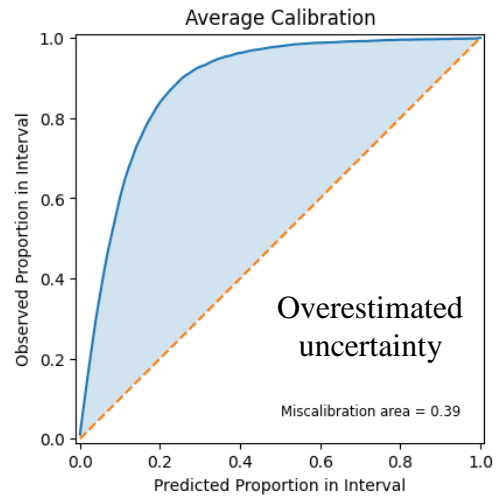
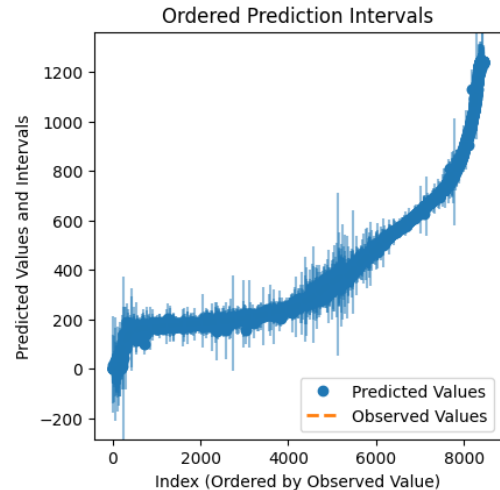
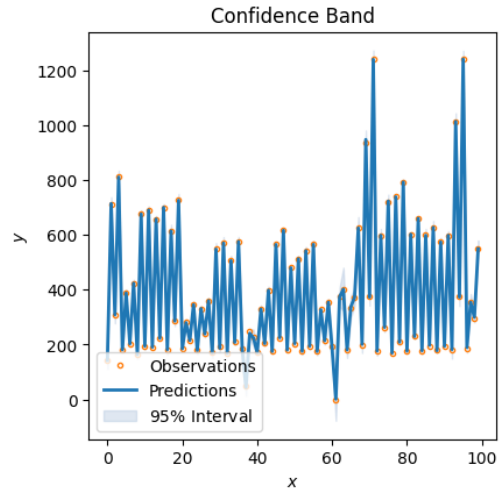


Recalibrate
→

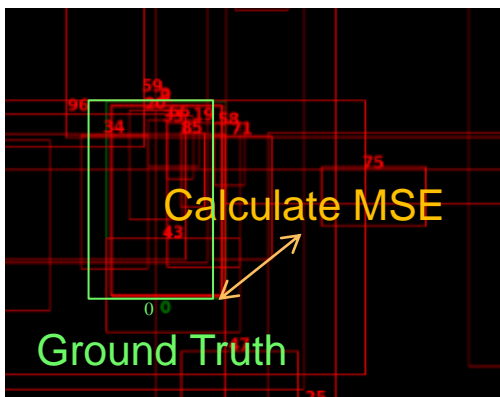
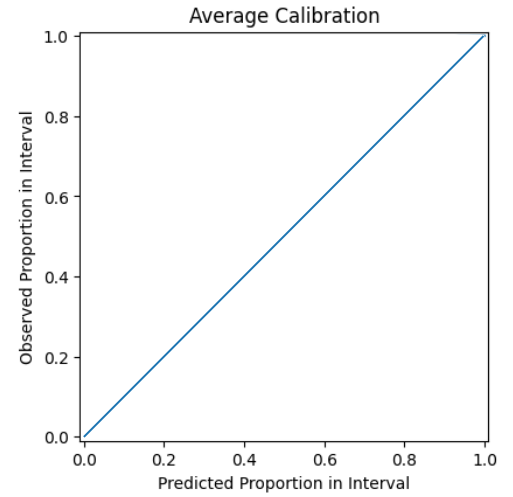


Calibration

Is the predicted uncertainty well-calibrated?



Recalibrate



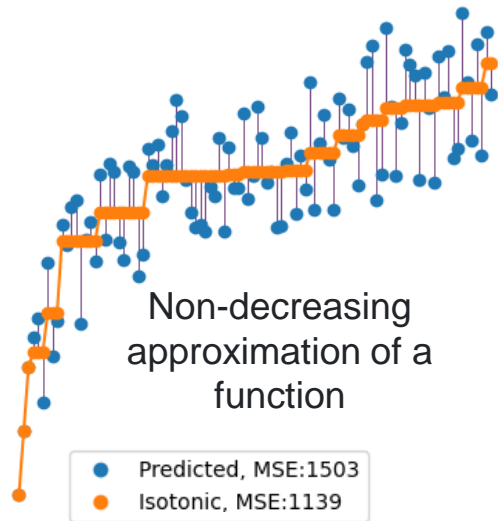
Prediction allocation via nearest-neighbor rather than thresholding:

- › For every **label**, a **corresponding box** is present.
- › Usually the one with the **highest classification score** is designated.
- › Does not necessarily correspond to the **ground truth** based on **localization**.

Calibration

Calibration Methods

Isotonic Regression



From one model to a model:

Per-coordinate

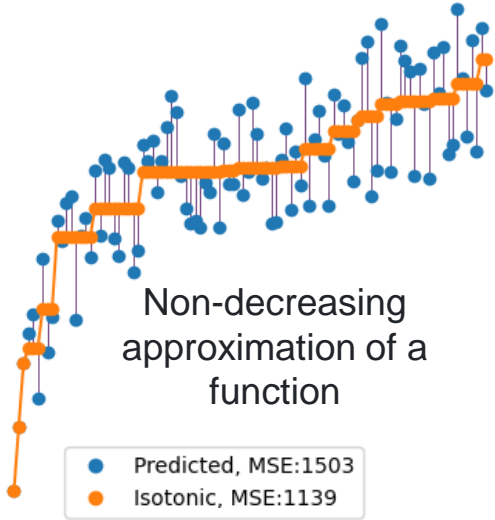
Per-class

Per-coordinate + per-class

Calibration

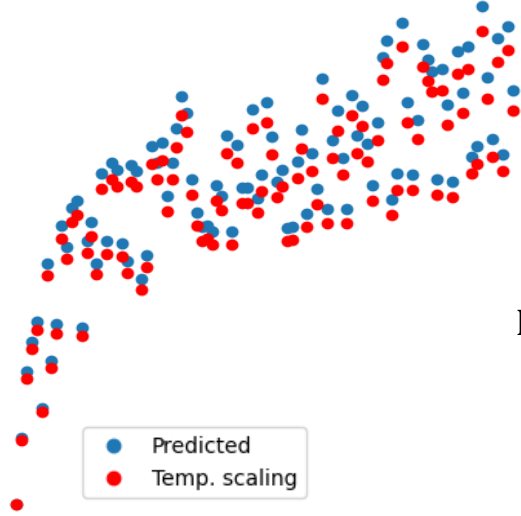
Calibration Methods

Isotonic Regression



From one model to a model:
Per-coordinate
Per-class
Per-coordinate + per-class

Factor Scaling



$$\mathcal{N}(\mu, s \cdot \sigma)$$

$$\Delta_i = |y_i^* - \mu_i| \text{ Residuals}$$

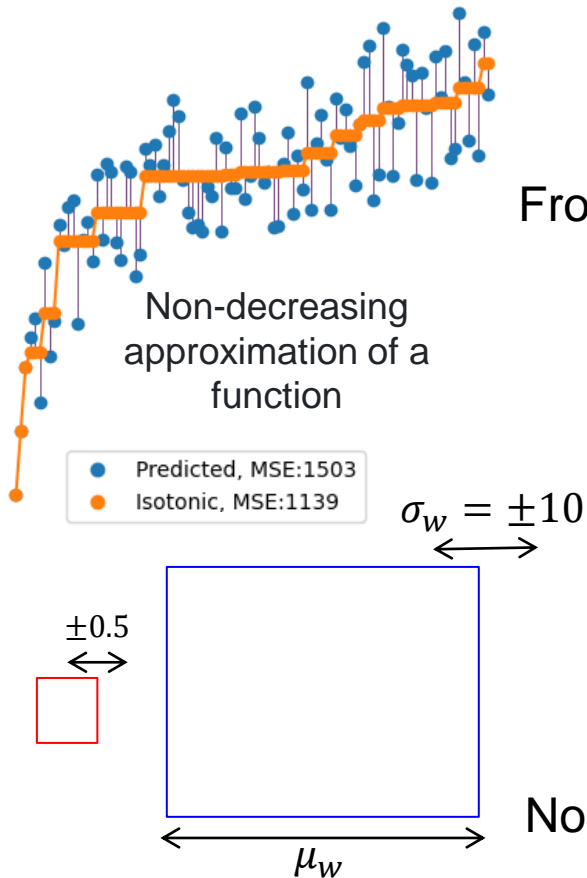
$$RMSE_{unc} = \sqrt{\frac{1}{N} \sum_{i=1}^N (\Delta_i - \sigma_i)^2}$$

$$MAE_{unc} = \frac{1}{N} \sum_{i=1}^N |\Delta_i - \sigma_i|$$

Calibration

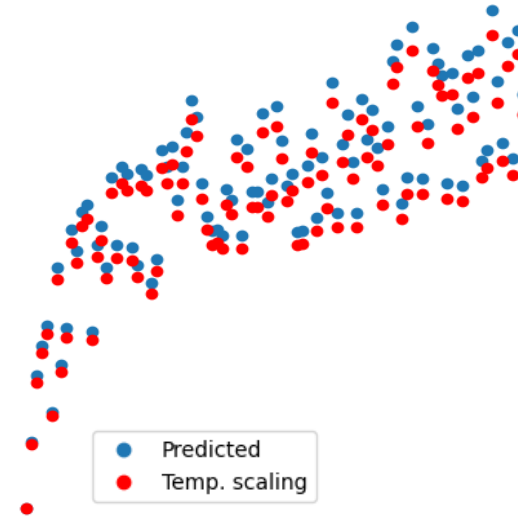
Calibration Methods

Isotonic Regression



From one model to a model:
Per-coordinate
Per-class
Per-coordinate + per-class

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$$\Delta_i = |y_i^* - \mu_i| \text{ Residuals}$$

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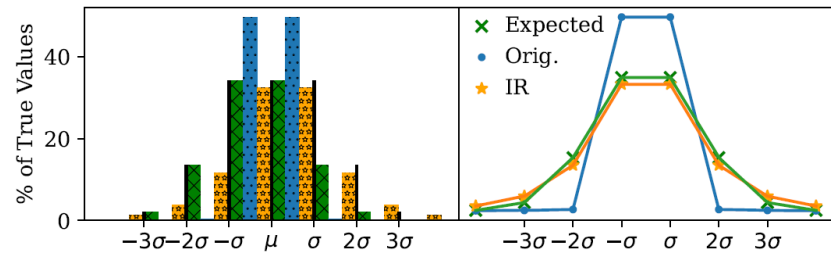
$$\text{MAE}_{unc} = \frac{1}{N} \sum_{i=1}^N |\Delta_i - \sigma_i|$$

Relative Calibration

Normalize Δ_i, σ_i with the corresponding width and height before calibration

Results

Calibration Methods

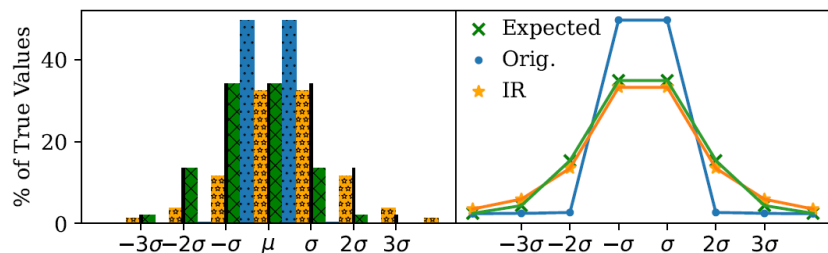


Distribution of the localization error; expected is a Gaussian

Uncertainty is mis-calibrated

Results

Calibration Methods



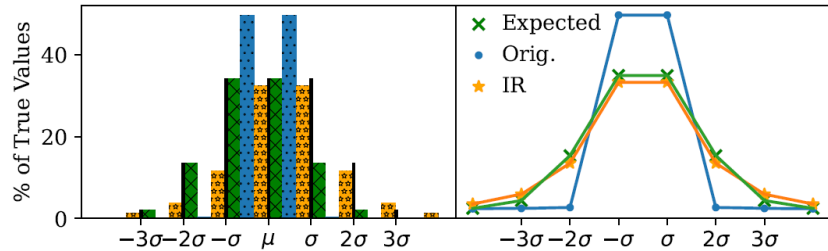
Distribution of the localization error; expected is a Gaussian

Method	KITTI				BDD			
	RMSUE↓	ECE↓	NLL↓	Sharp↓	RMSUE↓	ECE↓	NLL↓	Sharp↓
Uncalibrated	13.0 ± 0.0	0.384 ± 0.000	3.22 ± 0.0	14.9 ± 0.0	15.1 ± 0.1	0.323 ± 0.000	3.69 ± 0.0	17.22 ± 0.0
FS NLL	5.0 ± 0.3	0.194 ± 0.021	2.51 ± 0.1	4.7 ± 0.5	9.4 ± 0.4	0.217 ± 0.008	3.46 ± 0.0	9.72 ± 0.4
FS MAUE	4.6 ± 0.2	0.047 ± 0.001	3.14 ± 0.4	2.5 ± 0.0	7.5 ± 0.3	0.026 ± 0.001	4.72 ± 0.2	4.28 ± 0.0
FS RMSUE	4.6 ± 0.2	0.088 ± 0.003	2.79 ± 0.2	3.0 ± 0.0	7.6 ± 0.3	0.074 ± 0.000	6.43 ± 0.3	3.21 ± 0.0
Rel. FS RMSUE	7.2 ± 0.1	0.306 ± 0.002	2.74 ± 0.0	8.3 ± 0.1	8.5 ± 0.3	0.175 ± 0.003	3.50 ± 0.1	8.06 ± 0.1
Abs. IR	4.5 ± 0.2	0.032 ± 0.001	3.15 ± 0.3	2.5 ± 0.0	7.5 ± 0.3	0.027 ± 0.001	4.60 ± 0.1	4.09 ± 0.0
Abs. IR CL	4.4 ± 0.2	0.029 ± 0.001	2.86 ± 0.2	2.7 ± 0.0	7.4 ± 0.3	0.026 ± 0.001	4.39 ± 0.1	4.23 ± 0.0
Abs. IR PCo	4.5 ± 0.2	0.032 ± 0.001	3.03 ± 0.2	2.6 ± 0.0	7.5 ± 0.3	0.027 ± 0.001	4.57 ± 0.2	4.11 ± 0.0
Abs. IR PCo CL	4.3 ± 0.2	0.028 ± 0.000	2.70 ± 0.1	2.9 ± 0.0	7.4 ± 0.3	0.025 ± 0.001	4.36 ± 0.1	4.33 ± 0.0
Rel. IR	4.5 ± 0.2	0.027 ± 0.001	3.06 ± 0.3	2.5 ± 0.0	7.4 ± 0.3	0.018 ± 0.001	4.52 ± 0.1	4.07 ± 0.0
Rel. IR CL	4.4 ± 0.2	0.026 ± 0.001	2.78 ± 0.2	3.1 ± 0.4	7.3 ± 0.3	0.017 ± 0.000	4.29 ± 0.1	4.24 ± 0.0
Rel. IR PCo	4.5 ± 0.2	0.027 ± 0.001	3.03 ± 0.2	2.5 ± 0.1	7.4 ± 0.3	0.018 ± 0.000	4.49 ± 0.1	4.08 ± 0.0
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Uncertainty is mis-calibrated

Results

Calibration Methods



Distribution of the localization error; expected is a Gaussian

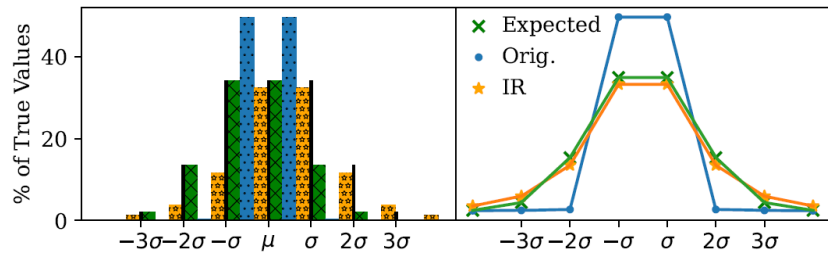
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Uncertainty is mis-calibrated

Our FS losses outperform SOTA

Results

Calibration Methods



Distribution of the localization error; expected is a Gaussian

Method	KITTI				BDD			
	RMSUE↓	ECE↓	NLL↓	Sharp↓	RMSUE↓	ECE↓	NLL↓	Sharp↓
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Rel. FS RMSUE	7.2 ± 0.1	0.306 ± 0.002	2.74 ± 0.0	8.3 ± 0.1	8.5 ± 0.3	0.175 ± 0.003	3.50 ± 0.1	8.06 ± 0.1
Abs. IR	4.5 ± 0.2	0.032 ± 0.001	3.15 ± 0.3	2.5 ± 0.0	7.5 ± 0.3	0.027 ± 0.001	4.60 ± 0.1	4.09 ± 0.0
Abs. IR CL	4.4 ± 0.2	0.029 ± 0.001	2.86 ± 0.2	2.7 ± 0.0	7.4 ± 0.3	0.026 ± 0.001	4.39 ± 0.1	4.23 ± 0.0
Abs. IR PCo	4.5 ± 0.2	0.032 ± 0.001	3.03 ± 0.2	2.6 ± 0.0	7.5 ± 0.3	0.027 ± 0.001	4.57 ± 0.2	4.11 ± 0.0
Abs. IR PCo CL	4.3 ± 0.2	0.028 ± 0.000	2.70 ± 0.1	2.9 ± 0.0	7.4 ± 0.3	0.025 ± 0.001	4.36 ± 0.1	4.33 ± 0.0
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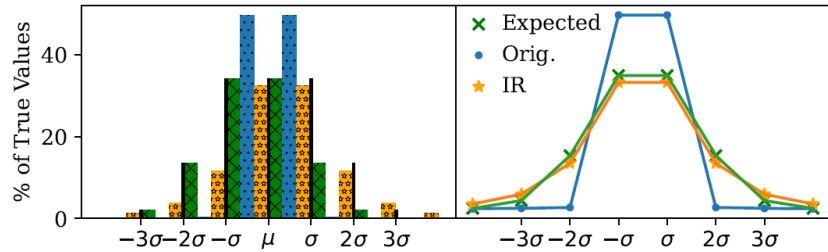
Uncertainty is mis-calibrated

Our FS losses outperform SOTA

Per-coordinate and per-class calibration IR outperforms other methods

Results

Calibration Methods



Distribution of the localization error; expected is a Gaussian

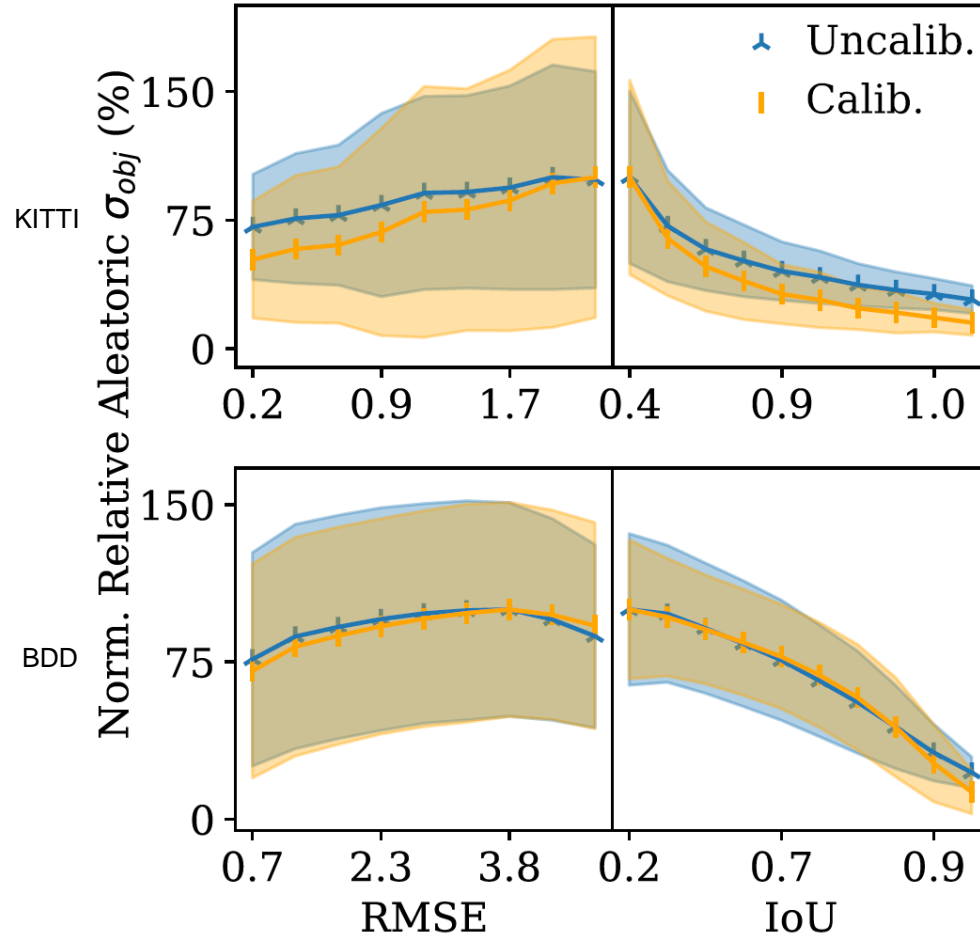
Uncertainty is mis-calibrated
Our FS losses outperform SOTA
Per-coordinate and per-class calibration IR
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Relative calibration affects small objects

Method	KITTI				BDD			
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	Uncalibrated		Absolute Calib.		Relative Calib.	
	ECE	NLL	ECE	NLL	ECE	NLL
ALL	0.384 ± 0.000	3.22 ± 0.01	0.033 ± 0.000	3.15 ± 0.42	0.031 ± 0.000	2.93 ± 0.18
Small	0.383 ± 0.000	2.95 ± 0.06	0.049 ± 0.005	4.54 ± 2.59	0.029 ± 0.002	3.16 ± 1.36
Medium	0.382 ± 0.000	3.00 ± 0.00	0.039 ± 0.001	2.55 ± 0.12	0.033 ± 0.001	2.42 ± 0.09
Large	0.387 ± 0.000	3.79 ± 0.00	0.056 ± 0.002	3.45 ± 0.15	0.041 ± 0.001	3.73 ± 0.22

Results

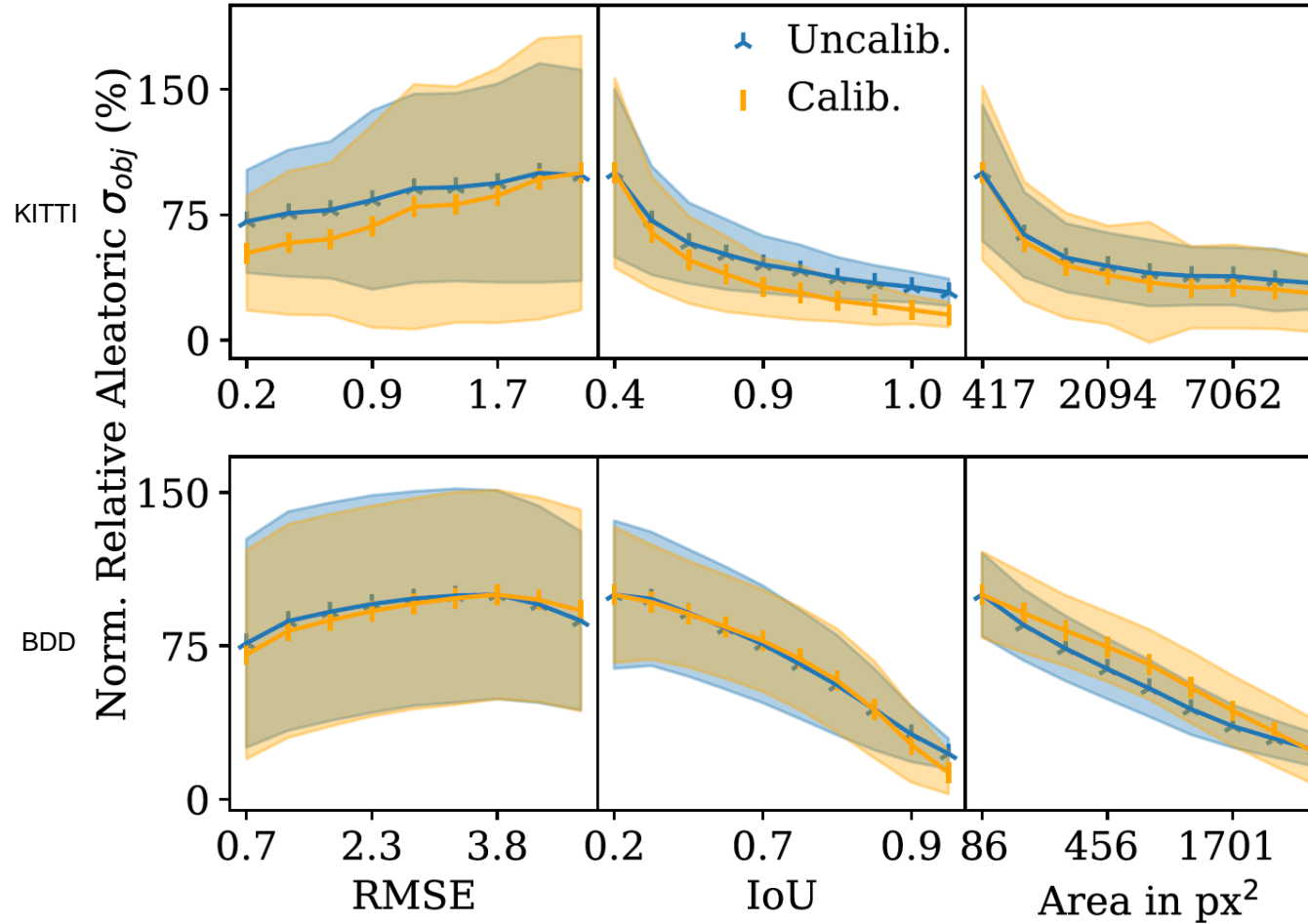
What correlations exist between the data and the uncertainty?



*Quantile-based binning, normalized by bin with highest uncertainty

Results

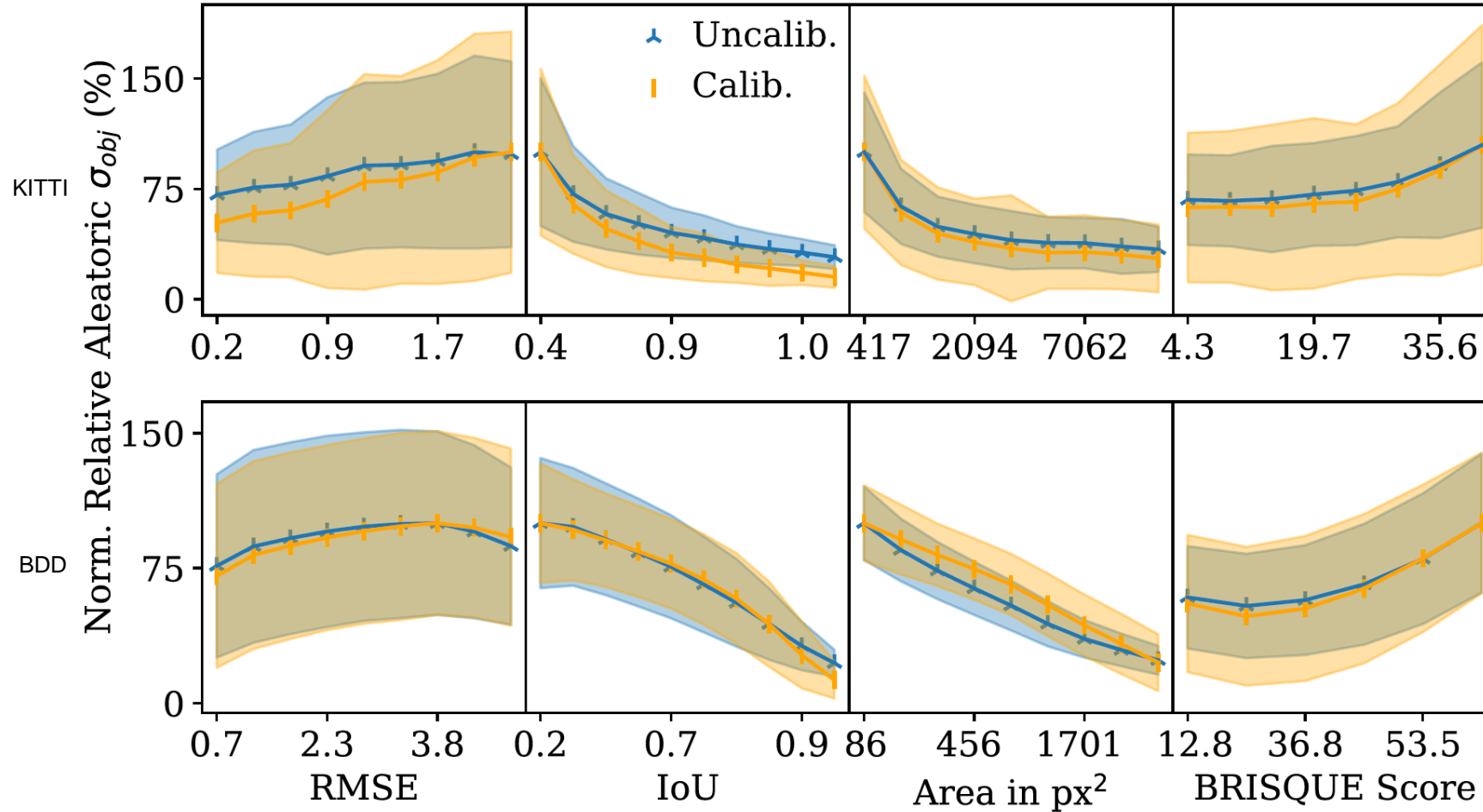
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Results

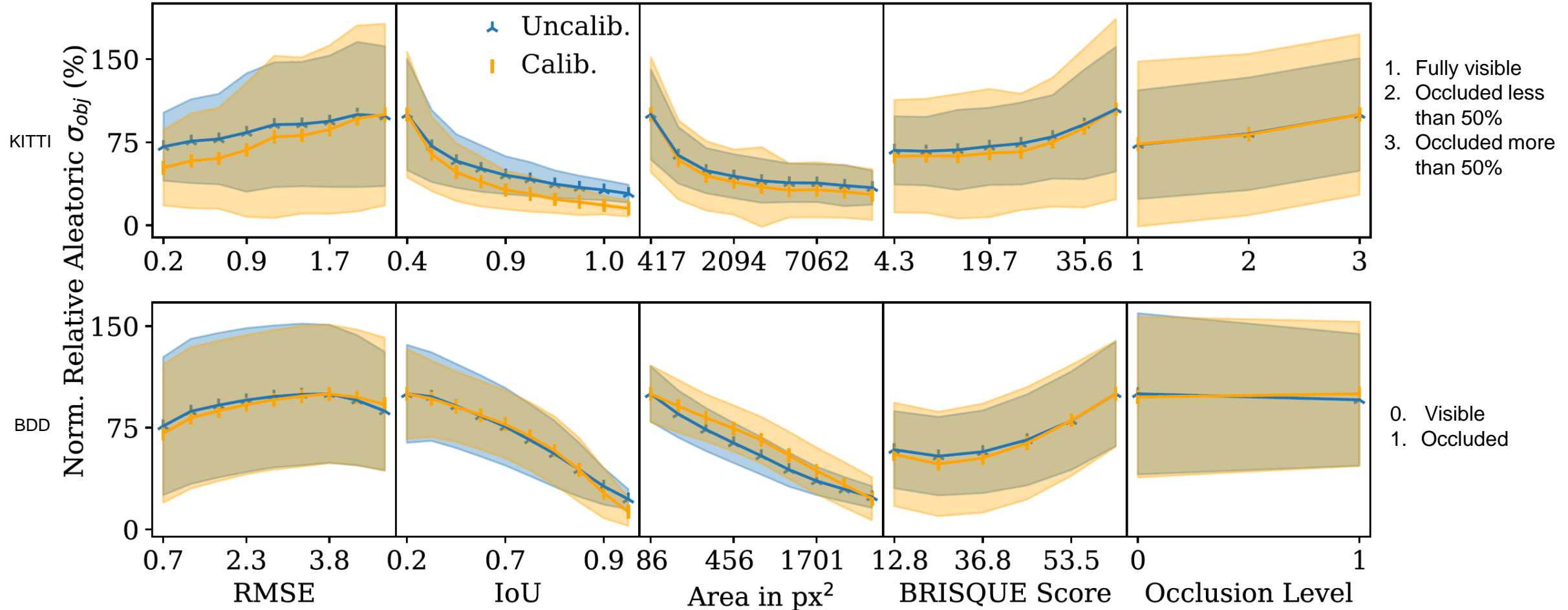
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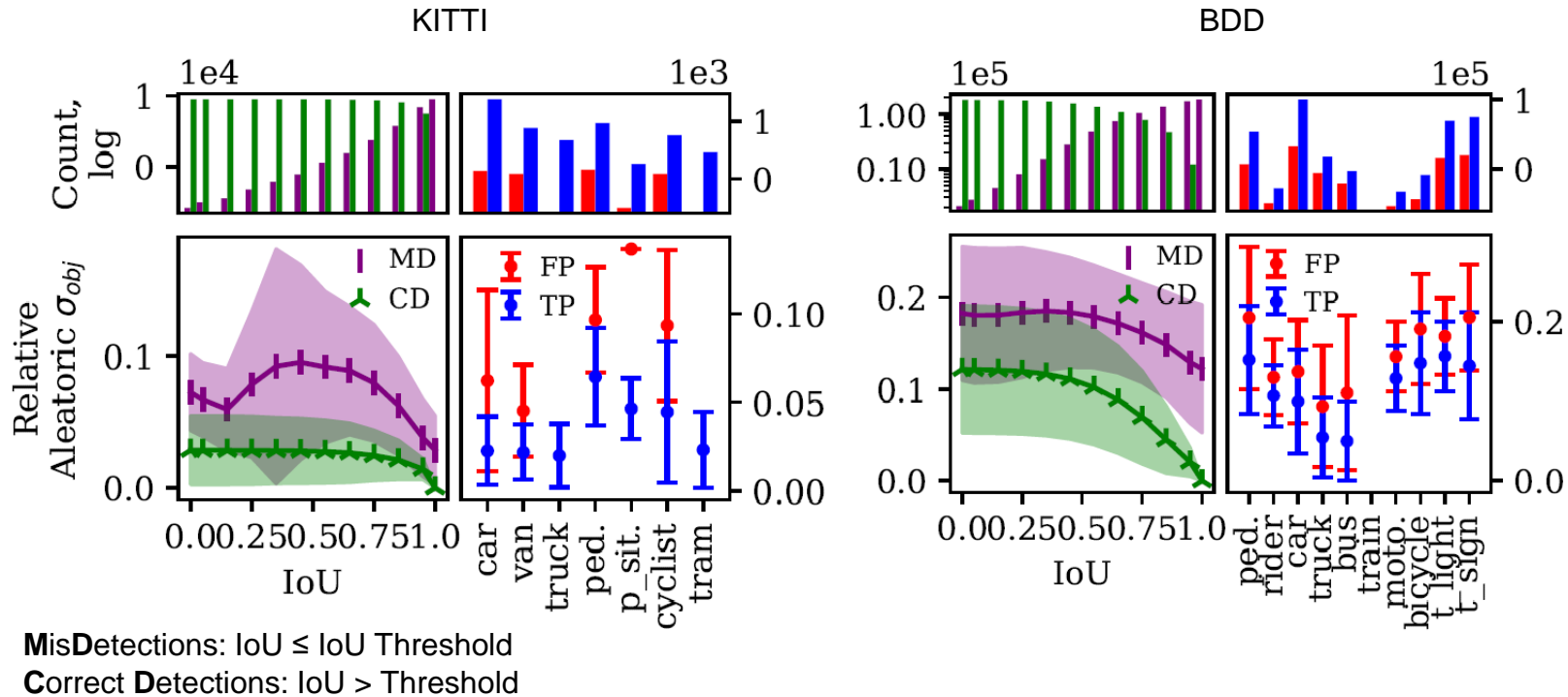
What correlations exist between the data and the uncertainty?



*Quantile-based binning, normalized by bin with highest uncertainty

Results

Correlation Analysis – Misdetections



*Negative correlation with detection accuracy
Possible thresholding via aleatoric uncertainty*

Summary

Loss attenuation for
EfficientDet with
**increased localization
performance.**

Summary



Loss attenuation for EfficientDet with **increased localization performance**.

Two decoding methods:

- › **Exact** and **fast** propagation.
- › **Generalize** for any non-linear functions in regression networks, for **different equations** and **distributions**.

Summary



Loss attenuation for **EfficientDet** with **increased localization performance**.

Two decoding methods:

- › **Exact** and **fast** propagation.
- › **Generalize** for any non-linear functions in regression networks, for **different equations** and **distributions**.

Extension of calibration methods: **Relative Per-coordinate, per-class calibration** with isotonic regression produces **well-calibrated uncertainties**.

Summary



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Paper



Appendix

State of the Art

Open Questions

- › How is the **output distribution** $N(\mu, \sigma^2)$ propagated through **non-linear** functions?
- › Is the predicted **uncertainty well-calibrated**?
- › What **correlations** exist between the **data** and **uncertainty**?

State of the Art

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- › **Incorrect** propagation, **no mention, only** used during **training** [1,2,3,4].
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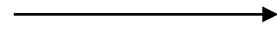
Missing propagation approach

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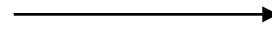
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- › **Occlusion**[4,11,12].
- › **Distance** in LiDAR data[11,12].
- › **Not** with detection accuracy [11,12].

*Missing propagation approach
Calibration not adapted to localization
Uncertainty still unclear*

Appendix

Propagation Methods

Normalizing Flows

A **normalizing flow** is a **transformation** of a **distribution** via a **sequence** of invertible and differentiable **mappings** [13]. *

$$h = e^{\hat{h}} h_a$$

$$g_1(\mathbf{y}) = \exp(\mathbf{y})$$

$$g_2(\mathbf{y}) = c\mathbf{y} \text{ and } c \in \mathbb{R}$$

$$h = g_2 \circ g_1(\hat{h}) \text{ with } c = h_a$$

* $p_{\mathbf{Y}}(\mathbf{y}) = p_{\mathbf{Z}}(\mathbf{f}(\mathbf{y})) |\det Df(\mathbf{y})|$
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Log-Normal Approach

If \hat{h} follows a **normal** distribution, then $e^{\hat{h}}$ follow a **log-normal** distribution [14].

$$f(x; \mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma(x)}} e^{-\frac{[\log(x)-\mu]^2}{2\sigma^2}}$$

$$E[e^{\hat{h}}] = e^{\mu_{\hat{h}} + \frac{\sigma_{\hat{h}}^2}{2}}$$

$$\text{Var}[e^{\hat{h}}] = \left[e^{\sigma_{\hat{h}}^2} - 1 \right] e^{2\mu_{\hat{h}} + \sigma_{\hat{h}}^2}$$

$$\mu_h = E[e^{\hat{h}}] \cdot h_a$$

$$\sigma_h^2 = \text{Var}[e^{\hat{h}}] \cdot h_a^2$$

Appendix

Calibration Example Analysis

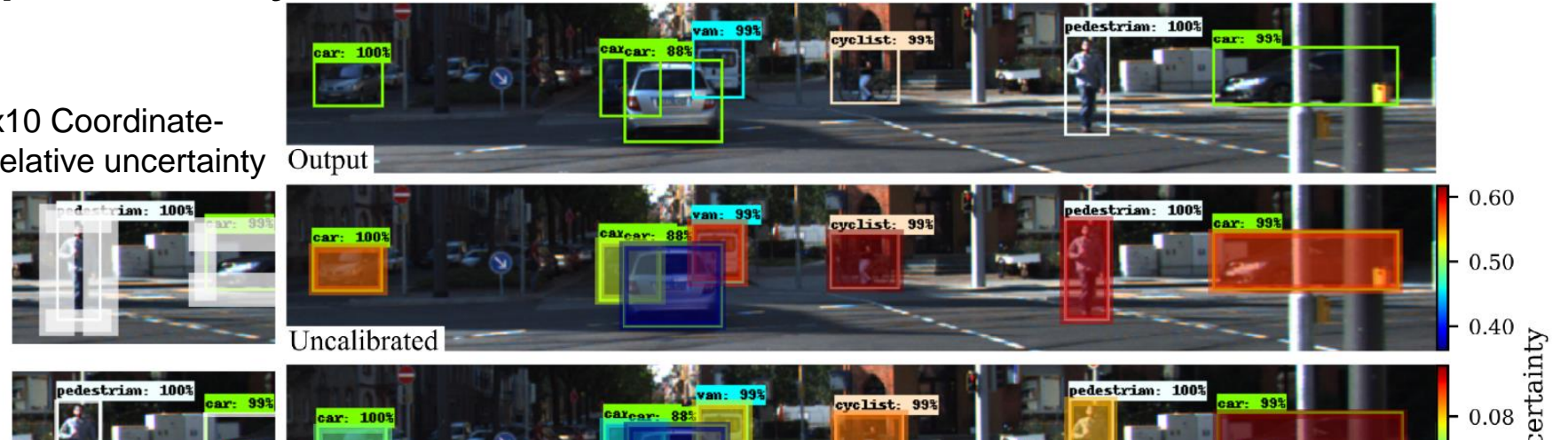


Appendix

Calibration Example Analysis

Calibration reduces uncertainty

x10 Coordinate-
relative uncertainty



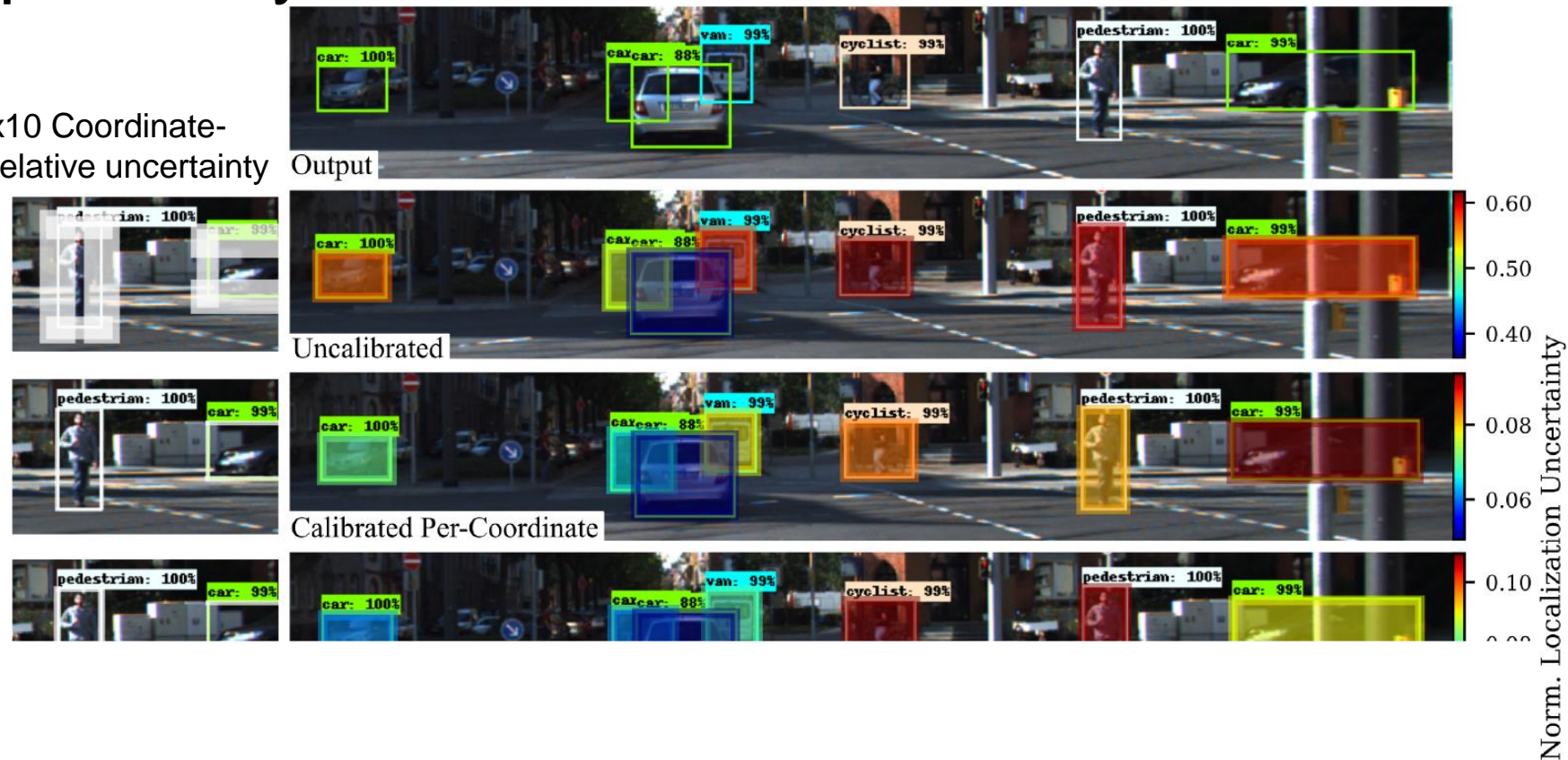
Appendix

Calibration Example Analysis

Calibration reduces uncertainty

Per-class calibration shifts uncertainty towards classes with lower performance

x10 Coordinate-
relative uncertainty



Appendix

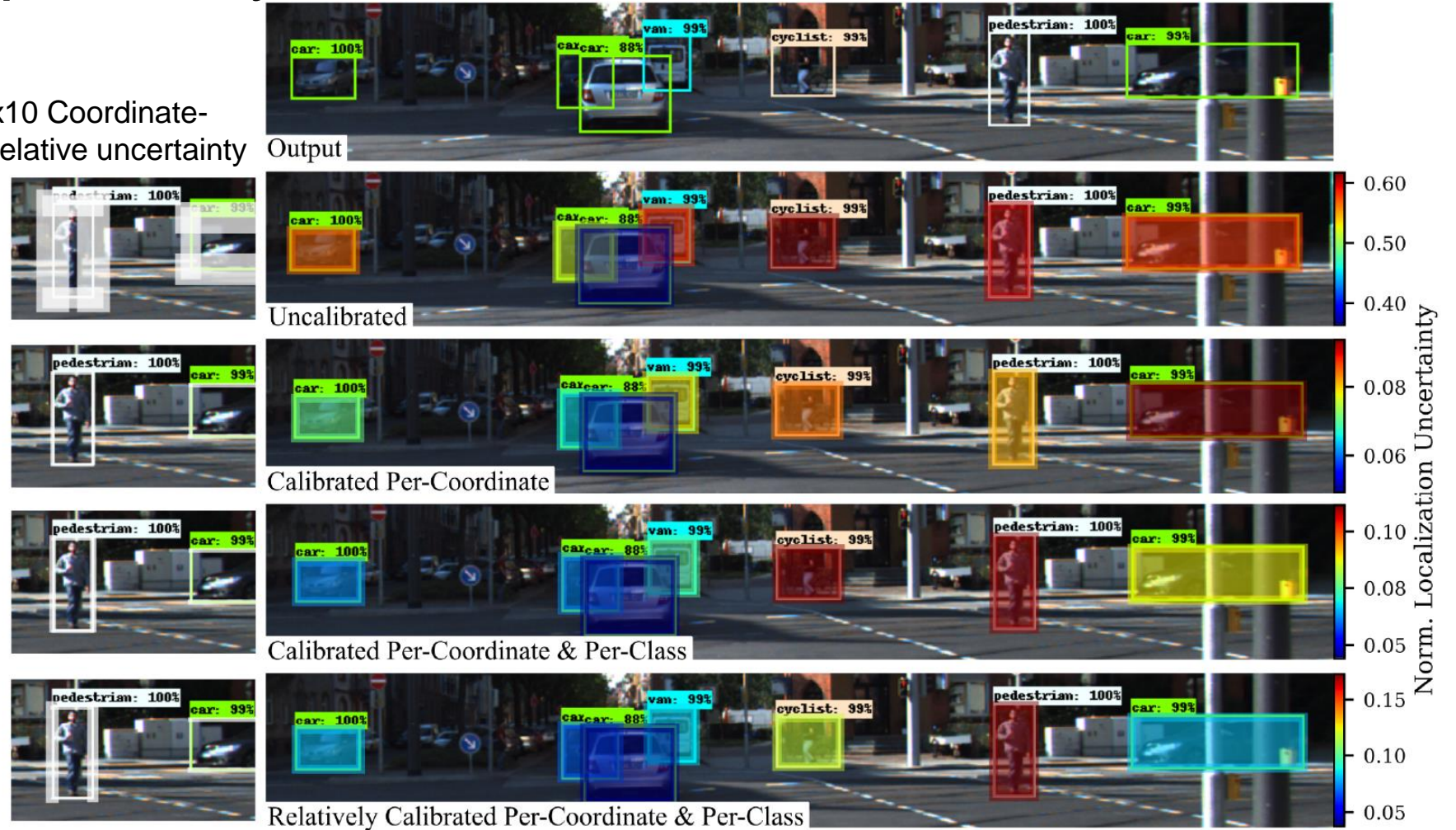
Calibration Example Analysis

Calibration reduces uncertainty

Per-class calibration shifts uncertainty towards classes with lower performance

Relative calibration considers the effect of the different aspect ratios






x10 Coordinate-
relative uncertainty



Appendix

BRISQUE Score

The Blind/Referenceless Image Spatial Quality Evaluator (BRISQUE) is a no-reference image quality assessment algorithm.

Original Image	JPEG2K Compression	Heavy Compression	Gaussian Noise	Median Blur
				
26.8286	30.7417	33.0692	79.8751	72.7044

BRISQUE:

- Is based on statistical models of natural image features, which are extracted from the image using a set of spatial and transform domain operators. The operators include discrete cosine transform (DCT), discrete wavelet transform (DWT), and local binary patterns (LBP), among others.
- Computes a score from the set of feature vectors that represents the degree of naturalness of the image, which correlates with the subjective quality of the image.
- Is a machine learning model, typically a Gaussian process or a support vector regression (SVR) model, trained on a large dataset of natural images TID2008, which were annotated with mean opinion scores (MOS) obtained from subjective experiments where human observers rated the perceived image quality.
- Ranges from 0 to 100, with higher scores indicating lower image quality and lower scores indicating higher image quality.

Appendix

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